VII. *Concerning the Proportion of* Mathematical Points *to each other. By the Honourable* Francis Robartes *Esq; Vice-President of the* Royal Society.

IT has heretofore pass'd for a current Maxime, That all Infinites are equal.

Divines and Metaphysicians have not scrupled to ground many of their Arguments on that Foundation.

The Position nevertheless is certainly erroneous, as Dr. *Halley* abundantly has shown in the *Philosophical Transaction* for *October* 1696. He there gives divers Instances of infinite quantities which are in a determinate finite proportion one to another, and some infinitely greater one than another.

The like may be observ'd of infinitely small quantities (*viz.*) Mathematical Points,) as the following Propositions will make appear.

PROP. I. *Fig.* 1.

The Points of contact between Circles and their Tangents are in Subduplicate proportion to the Diameters of the Circles.

Let two Circles *a d c b*, *e f b g*, touch one another from within at the point *a*. Draw the Tangent *p a q*, and parallel to it the line *m n*. From the point *a* draw the Diameter *a c*.

Let

Let ac the Diameter of the greater Circle be equal to R, and ab the Diameter of the leſſer Circle be equal to S.

Let dh the Chord of the Arch dab be equal to z, and fg the Chord of the Arch fag be equal to y, and let the Abſciſs ak be equal to x.

If he Line mn be ſuppoſed to move till it becomes coincident with the Tangent paq, the nature of a Circle will always give the following Æquations.

$$zz = 4Rx - 4xx.$$
$$yy = 4Sx - 4xx.$$

When the Line is arrived at the Tangent, z and y will become the two Points of Contact, and then $zz=4Rx$ and $yy=4Sx$. ($4xx$ being laid aſide as Heterogeneous to the reſt of the Æquation, by reaſon of x being become infinitely little) Therefore

$$zz . yy :: 4Rx . 4Sx :: R . S.$$

Therefore $z . y :: \sqrt{R} . \sqrt{S}$. *Q. E. D.*

PROP. II. *Fig. 2.*

The Point of Contact between a Sphere and a Plane is infinitely greater than that between a Circle and a Tangent.

Let a be the Point of Contact between the Sphere $adqf$ and the Plane bc. About the Sphere deſcribe the Cylinder $npgm$.

Draw kh to repreſent a Circle parallel to the Plane. Let the Circle be ſuppos'd to move, till it becomes coincident with the Plane. The Cylindrical Surface $khgm$ will always be equal (according to *Archimedes*) to the Spherical Surface daf.

Now

Now when these Surfaces become infinitely small, one terminates in the Point of Contact, and the other in the Periphery of the Base of the Cylinder. Therefore the Point of Contact is equal to the Periphery of the Base of the Cylinder (equal to a Periphery which has the same Diameter as the Sphere) and by consequence is infinitely greater than any point of Contact between a Circle and a Tangent. *Q. E. D.*

PROP. III.

The Points of Contact by Spheres of different Magnitude are to one another as the Diameters of the Spheres.

For by the second Proposition the Points of Contact are equal to the Peripheries of such Diameters, whose proportion is the same as the Diameters.

Q. E. D.

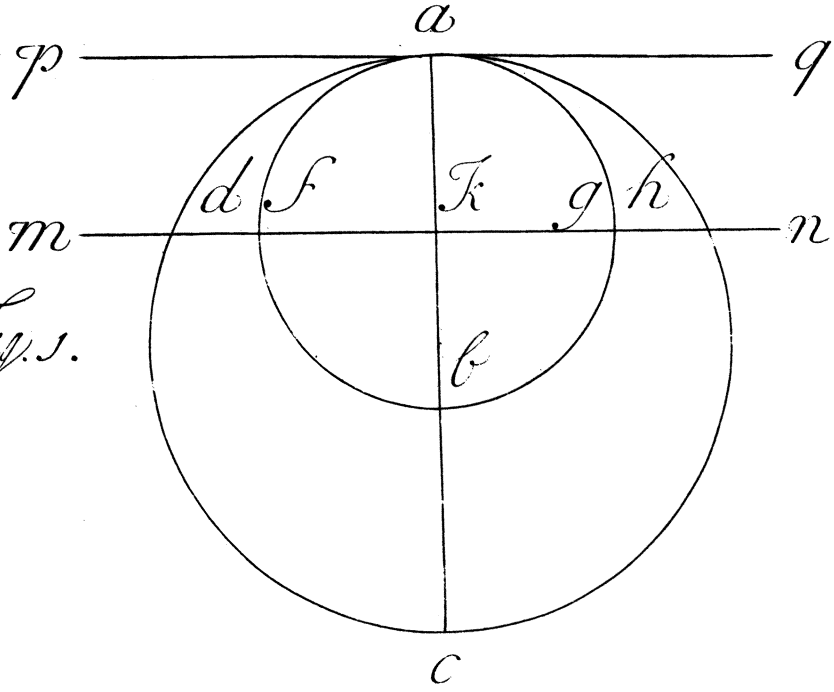

Fig. 1.

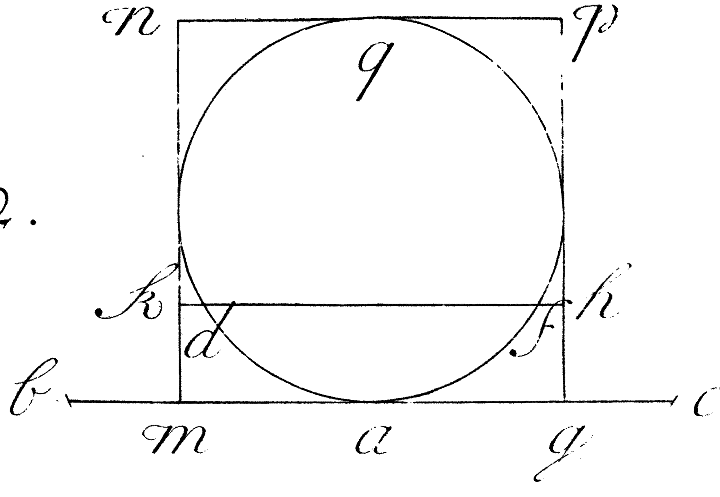

Fig: 2.